Met O 3 Technical Note No.

# Comparison of the Harmonics Representing Seasonal Variation in Air Temperature over the United Kingdom for 1901-30 and 1941-70

by

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# Comparison of the Harmonics Representing Seasonal Variation in Air Temperature over the United Kingdom for 1901-30 and 1941-70

## 1. Introduction

The purpose of this study is to examine whether temporal changes have occurred in the seasonal variation of maximum and minimum temperatures in the United Kingdom. This is achieved by comparing the characteristics of the first two annual harmonics describing the seasonal variation in temperatures for the periods 1901-30 and 1941-70.

## 2. Previous work

Craddock (1956) has studied the first and second harmonics of the seasonal variation in temperatures using monthly mean temperatures at 160 stations in the British Isles. The monthly means were averaged over the period 1921-50. His results indicate a regular pattern across the country to the phase and amplitude of the first harmonic and also to the amplitude of the second harmonic. Also evident is a regular variation over the country of the difference in dates of peak values given by the first and second harmonics. Craddock relates this last result to spatial differences in the characteristics of temperature changes through the year.

In this paper maximum and minimum temperatures are treated separately and the main emphasis is on identifying changes in the harmonics over the two thirty-year periods.

## 3. Data

Five widely scattered stations having almost unbroken daily maximum and daily minimum temperatures since 1900 were selected for the analysis. These five are Plymouth, Oxford, Armagh, Durham and Gordon Castle and their locations are shown in fig 1. The Plymouth temperature series are for the Hoe until 1930 and for Mountbatten since 1930. The former site is relatively sheltered whereas the site at Mountbatten is in an exposed position on the crest of a hill. A separate study by Smith (1978) suggested there is evidence for inhomogeneity of observations in these series which may be due to the change of site. October 1956 minimum temperatures for Gordon Castle are missing and so were excluded from the analysis.

## 4. Method of analysis

Future reference to "maxima" and "minima" will denote the highest and lowest daily values. "Peak value" will refer to the highest temperature attained by an harmonic in the regression model described below.

For each station, maxima and minima were averaged for each day of the year over the periods 1901-30 and 1941-70. February 29th was omitted and so each series comprised 365 terms. Each term $y_t$ was assumed to follow an expression of the form

$$y_t = \bar{y} + a_1 \cos ct + b_1 \sin ct + a_2 \cos 2ct + b_2 \sin 2ct + \ldots$$

where $\bar{y}$ is the mean,

$a_i, b_i$ are the components of the $i^{th}$ annual harmonic,

$c = \frac{2\pi}{365}$

and $t$ is measured in days from midnight on December 31st.

It was established that the first two harmonics accounted for over 95% of the variance of the maxima and minima series and therefore harmonics of higher order than the second were ignored for the purpose of this study.

The components of the first and second harmonics were estimated using a regression program in the BMDP statistical package (Dixon (1975)). The amplitudes $A_i$ and phases $\phi_i$ were calculated from the components, where

$$A_i = (a_i^2 + b_i^2)^{\frac{1}{2}}$$

$$\phi_i = \tan^{-1}\left(\frac{b_i}{a_i}\right)$$

The date of the peak value of the first harmonic and the date of the summer peak value of the second harmonic were determined from $\phi$.

## 5. Results

Table 1 presents annual means and variances of the daily means for the different periods. For both maxima and minima, the 1941-70 means are about 0.3°C higher than 1901-30 values, the exception being for Plymouth maxima. Study of the daily series for each station indicated that values in January and February were in fact lower during the later period; the overall increase in the annual mean for 1941-70 is therefore due to warming in months other than January and February. The variances also show increases between 1901-30 and 1941-70.

Figs 2-5 show, for the first harmonic in 1941-70, the variation across the country of the amplitude and of the date of peak value for maxima (figs 2 and 3) and for minima (figs 4 and 5). The isopleths are based on the pattern of variation observed by Craddock (1956) but it would be unwise to infer parameter values from figs 2-5 alone. The standard error of the amplitude is approximately 0.05°C and that of the date of the peak value about 0.5 days, with the standard error of the latter inversely proportional to the amplitude.

The differences in the amplitude and phase between the five stations agree with Craddock's results and will not be discussed here. Comparing fig 2 and fig 4 it is seen that the amplitude for maxima is 20-30% greater than for minima. The date of the peak value for maxima (fig 3) occurs 8-11 days earlier than for minima (fig 5). The result for the amplitudes is a measure of the greater annual range of maxima; the result for the dates of peak value is possibly due to a combination of two factors:

(i) greater incoming solar radiation received as January progresses without a corresponding decrease in outgoing night-time long-wave radiation - so maxima begin to rise earlier in the year than minima.

(ii) sea temperatures, which exert a stronger influence on minima than maxima in Winter, reach their lowest values relatively late in the Winter.

Table 2 gives various statistics relating to the first harmonics. The variance explained by the harmonic (expressed as a proportion of the variance of the series in the table) is equal to $A^2/2$ where A is the amplitude. The harmonic contributes about 98% of the total variance for maxima and only slightly less for minima. This proportion increases slightly between 1901-30 and 1941-70 for maxima and more substantially for minima. The date of peak value for maxima and minima remains relatively constant, the only statistically significant change (of four days) occurring for Plymouth minima.

Table 3 presents statistics for the second harmonics. The variance explained for maxima is less than 1% of the total and shows a decrease from 1901-30 to 1941-70. For minima the amount explained is about 3.5% in 1901-30 but drops to below 1% in 1941-70. The phase of the second harmonic has also altered considerably, particularly for maxima. The date of peak value for maxima occurs about one month later in the 1941-70 period and 11 days later for minima, compared to 1901-30.

Table 4 shows how the amplitudes and phases of the first two harmonics have altered relative to each other between 1901-30 and 1941-70. It is seen from column 1 that the amplitude of the second harmonic is reduced compared to that of the first harmonic in the later period, more especially for minima. The second column indicates that the differences in peak value dates between the two harmonics have changed substantially for maxima, by about 25 days. There is also a change in the same sense for minima but not as great as great as for maxima. These changes in the dates of peak value are due almost entirely to variations in the phase of the second harmonic, since, as shown earlier, variations in phase of the first harmonic have been negligible.

The results given in table 4 imply that:

(i) the seasonal variation in temperature more closely resembles a pure sine curve of period one year in 1941-70 than in 1901-30.

(ii) for the later period relative to the earlier period, the Winter has become more pronounced and the Summer extends later into the year, with a consequent shortening of the Autumn season.

Outcome (ii) is deduced from the relative change in phases of the two harmonics. Note that because of the small amplitude of the second harmonic in 1941-70 the effects of the phase change are correspondingly diminished. In support of (ii), more pronounced Winters are indeed evident over 1941-70 than for 1901-30 since January and February are colder and other months warmer in the later period relative to 1901-30. Furthermore, Clark (1979) has reported an increase in Central England October temperatures in the period since 1940 relative to earlier years, which agrees with the idea that Summer has extended later into the year for 1941-70.

## 6. Conclusion

The amplitudes and phases of the first two annual harmonics of maximum and minimum temperatures at five U.K. stations have been compared for the periods 1901-30 and 1941-70 and it has been found that:

(i) The amplitude of the first harmonics for maxima and for minima are greater over 1941-70 than over 1901-30. The opposite is true for the amplitudes of the second harmonic.

(ii) The phases of the first harmonic for maxima and for minima have remained unchanged. Those for the second harmonic have altered such that the date of the Summer peak value (and of other stationary points) is later in the more recent period.

## 7. Postscript

An harmonic analysis was carried out on monthly mean sea surface temperatures (SST) for 1901-30 and 1941-70. The area to which the SST relate roughly corresponds to the coastal waters of SW England. The intention of this brief analysis was to examine and compare fluctuations in the harmonics for SST with those observed for air temperatures over land. For SST the amplitudes of the first harmonic increased and of the second decreased for 1941-70 relative to 1901-30, while the dates of peak value of both harmonics increased by 3-4 days between 1901-30 and 1941-70.

The results for the amplitudes agree with those for air temperatures. The results for the phases are somewhat different but the changes that there are act in the same sense as found for air temperatures.

## 8. References


Clark, J. B. 1979 An investigation into recent October temperature trends over Central England. Weather, Vol.34, No.10, pp.374-383.

Craddock, J. M. 1956 The harmonic representation of the annual temperature variation in different parts of the British Isles. Meteorological Research Paper.No.970.

Dixon, W. J. 1975 Biomedical Computer Programs (BMDP). Manual date 1975. Program used was BMDP2R.

Smith, S. G. 1978 Long term optimum averaging periods for temperatures in the United Kingdom. Met O3.Technical Note.No.1.

# TABLE 1

## Summary data for each station and period

(i) Maxima

| | | Mean (deg C) | Variance (deg C)$^2$ |
|---|---|---|---|
| Oxford | 1901-30 | 13.68 | 28.14 |
| | 1941-70 | 13.89 | 31.14 |
| Plymouth | 1901-30 | 13.76 | 16.81 |
| | 1941-70 | 13.48 | 17.00 |
| Armagh | 1901-30 | 12.60 | 18.97 |
| | 1941-70 | 13.04 | 20.69 |
| Durham | 1901-30 | 11.85 | 23.43 |
| | 1941-70 | 12.38 | 26.22 |
| Gordon Castle | 1901-30 | 11.87 | 19.79 |
| | 1941-70 | 12.07 | 21.41 |

(ii) Minima

| | | | |
|---|---|---|---|
| Oxford | 1901-30 | 5.96 | 14.73 |
| | 1941-70 | 6.28 | 17.44 |
| Plymouth | 1901-30 | 7.66 | 12.09 |
| | 1941-70 | 7.89 | 12.20 |
| Armagh | 1901-30 | 5.39 | 11.22 |
| | 1941-70 | 5.68 | 12.77 |
| Durham | 1901-30 | 4.50 | 13.67 |
| | 1941-70 | 4.83 | 15.13 |
| Gordon castle | 1901-30 | 4.45 | 11.42 |
| | 1941-70 | 4.82 | 13.41 |

# TABLE 2

## Statistics for the first harmonic

(i) Maxima

| Station and period | | Components | | Amplitude | | Phase |
|---|---|---|---|---|---|---|
| | | sin | cos | Value | Variance as % of total | Date of peak value (day/month) |
| Oxford | 1901-30 | -2.27 | -7.06 | 7.42 | 97.8 | 19/7 |
| | 1941-70 | -2.49 | -7.41 | 7.82 | 98.2 | 20/7 |
| Plymouth | 1901-30 | -2.44 | -5.19 | 5.73 | 97.6 | 27/7 |
| | 1941-70 | -2.58 | -5.17 | 5.78 | 98.2 | 28/7 |
| Armagh | 1901-30 | -1.92 | -5.78 | 6.09 | 97.7 | 20/7 |
| | 1941-70 | -1.93 | -6.08 | 6.38 | 98.4 | 19/7 |
| Durham | 1901-30 | -2.41 | -6.31 | 6.76 | 97.5 | 22/7 |
| | 1941-70 | -2.55 | -6.70 | 7.17 | 98.1 | 22/7 |
| Gordon Castle | 1901-30 | -2.17 | -5.80 | 6.20 | 97.1 | 22/7 |
| | 1941-70 | -2.29 | -6.03 | 6.43 | 96.5 | 22/7 |

(ii) Minima

| | | | | | | |
|---|---|---|---|---|---|---|
| Oxford | 1901-30 | -2.35 | -4.72 | 5.27 | 94.3 | 28/7 |
| | 1941-70 | -2.64 | -5.19 | 5.82 | 97.2 | 28/7 |
| Plymouth | 1901-30 | -2.45 | -4.11 | 4.78 | 94.5 | 1/8 |
| | 1941-70 | -2.71 | -4.03 | 4.86 | 96.8 | 5/8 |
| Armagh | 1901-30 | -2.22 | -3.98 | 4.56 | 92.5 | 31/7 |
| | 1941-70 | -2.38 | -4.35 | 4.96 | 96.2 | 30/7 |
| Durham | 1901-30 | -2.49 | -4.42 | 5.07 | 94.0 | 31/7 |
| | 1941-70 | -2.70 | -4.72 | 5.43 | 97.4 | 31/7 |
| Gordon Castle | 1901-30 | -2.16 | -4.10 | 4.64 | 94.3 | 29/7 |
| | 1941-70 | -2.42 | -4.50 | 5.11 | 97.3 | 30/7 |

# TABLE 3

## Statistics for the second harmonic

(i) Maxima

| Station and period | | Components | | Amplitude | | Phase |
|---|---|---|---|---|---|---|
| | | Sin | cos | Value | variance as % of total | Date of summer peak (day/month) |
| Oxford | 1901-30 | 0.54 | 0.06 | 0.55 | 0.5 | 13/8 |
| | 1941-70 | 0.30 | -0.32 | 0.43 | 0.3 | 8/9 |
| Plymouth | 1901-30 | 0.46 | 0.07 | 0.47 | 0.7 | 11/8 |
| | 1941-70 | 0.19 | -0.14 | 0.24 | 0.2 | 2/9 |
| Armagh | 1901-30 | 0.52 | 0.14 | 0.54 | 0.8 | 8/8 |
| | 1941-70 | 0.24 | -0.26 | 0.36 | 0.3 | 8/9 |
| Durham | 1901-30 | 0.61 | 0.18 | 0.63 | 0.8 | 8/8 |
| | 1941-70 | 0.33 | -0.27 | 0.43 | 0.4 | 4/9 |
| Gordon castle | 1901-30 | 0.54 | 0.12 | 0.55 | 0.8 | 9/8 |
| | 1941-70 | 0.30 | -0.28 | 0.41 | 0.4 | 6/9 |

(ii) Minima

| | | | | | | |
|---|---|---|---|---|---|---|
| Oxford | 1901-30 | 0.77 | 0.54 | 0.94 | 3.0 | 29/7 |
| | 1941-70 | 0.48 | 0.16 | 0.50 | 0.7 | 7/8 |
| Plymouth | 1901-30 | 0.66 | 0.49 | 0.82 | 2.8 | 28/7 |
| | 1941-70 | 0.31 | 0.13 | 0.34 | 0.5 | 5/8 |
| Armagh | 1901-30 | 0.77 | 0.61 | 0.98 | 4.3 | 27/7 |
| | 1941-70 | 0.47 | 0.06 | 0.48 | 0.9 | 12/8 |
| Durham | 1901-30 | 0.85 | 0.47 | 0.97 | 3.4 | 1/8 |
| | 1941-70 | 0.41 | 0.07 | 0.42 | 0.6 | 11/8 |
| Gordon Castle | 1901-30 | 0.69 | 0.57 | 0.89 | 3.5 | 27/7 |
| | 1941-70 | 0.38 | 0.08 | 0.39 | 0.6 | 9/8 |

# TABLE 4

## Relationships between first and second harmonics

Key : $A_1, A_2$ - amplitudes of first and second harmonics

$D_1, D_2$ - dates of peak values for first and second harmonics

(i) Maxima

| | | $\frac{A_2}{A_1} \times 100\%$ | $D_2 - D_1$ (days) |
|---|---|---|---|
| Oxford | 1901-30 | 7 | 25 |
| | 1941-70 | 6 | 50 |
| Plymouth | 1901-30 | 8 | 15 |
| | 1941-70 | 4 | 36 |
| Armagh | 1901-30 | 9 | 19 |
| | 1941-70 | 6 | 51 |
| Durham | 1901-30 | 9 | 17 |
| | 1941-70 | 6 | 44 |
| Gordon Castle | 1901-30 | 9 | 18 |
| | 1941-70 | 6 | 46 |

(ii) Minima

| | | | |
|---|---|---|---|
| Oxford | 1901-30 | 18 | 1 |
| | 1941-70 | 9 | 10 |
| Plymouth | 1901-30 | 17 | -3 |
| | 1941-70 | 7 | 0 |
| Armagh | 1901-30 | 21 | -4 |
| | 1941-70 | 10 | 13 |
| Durham | 1901-30 | 19 | 1 |
| | 1941-70 | 8 | 11 |
| Gordon Castle | 1901-30 | 19 | -2 |
| | 1941-70 | 8 | 10 |

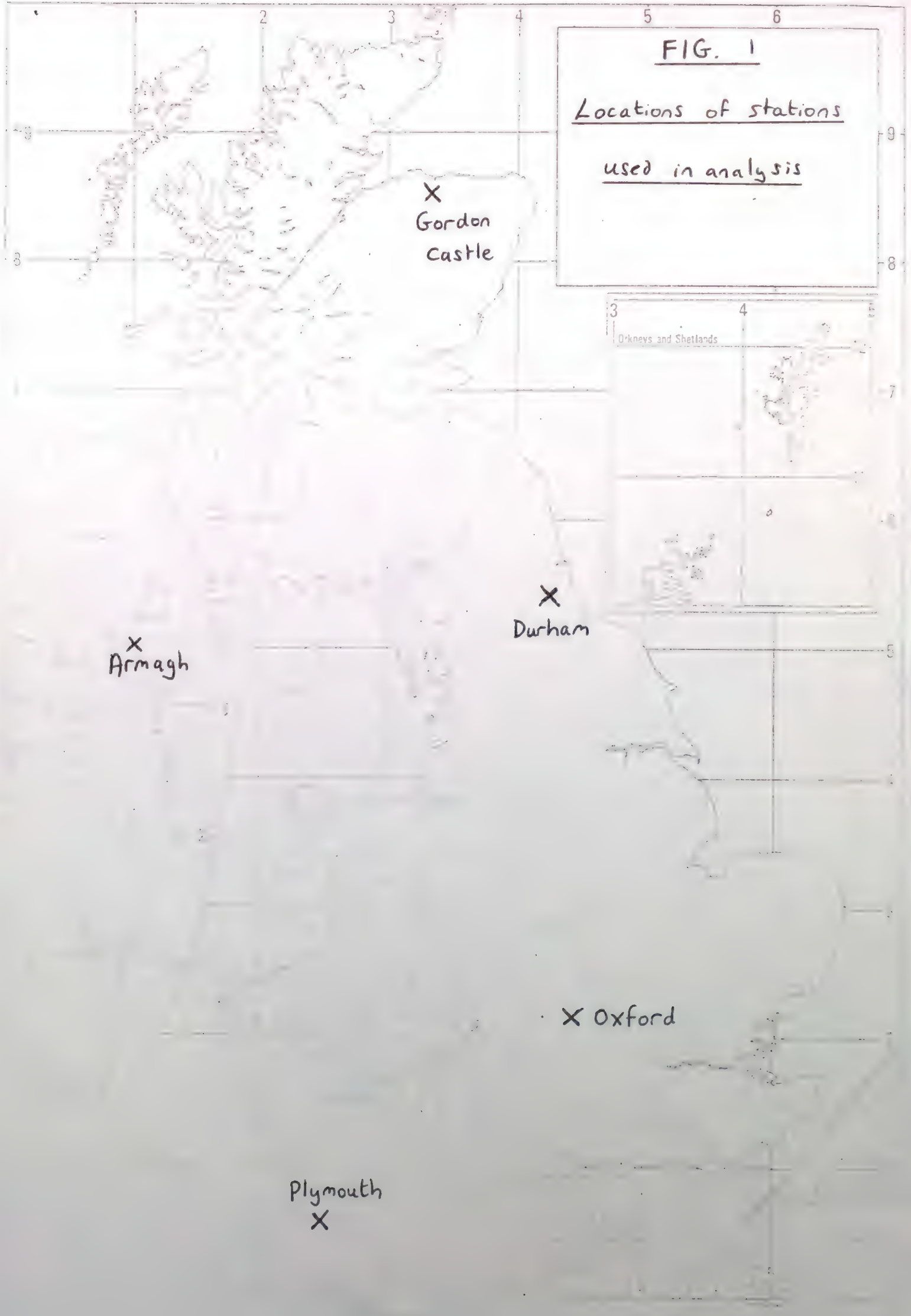
FIG. 1
Locations of stations
used in analysis
Gordon
Castle
Durham
Armagh
Oxford
Plymouth
Orkneys and Shetlands

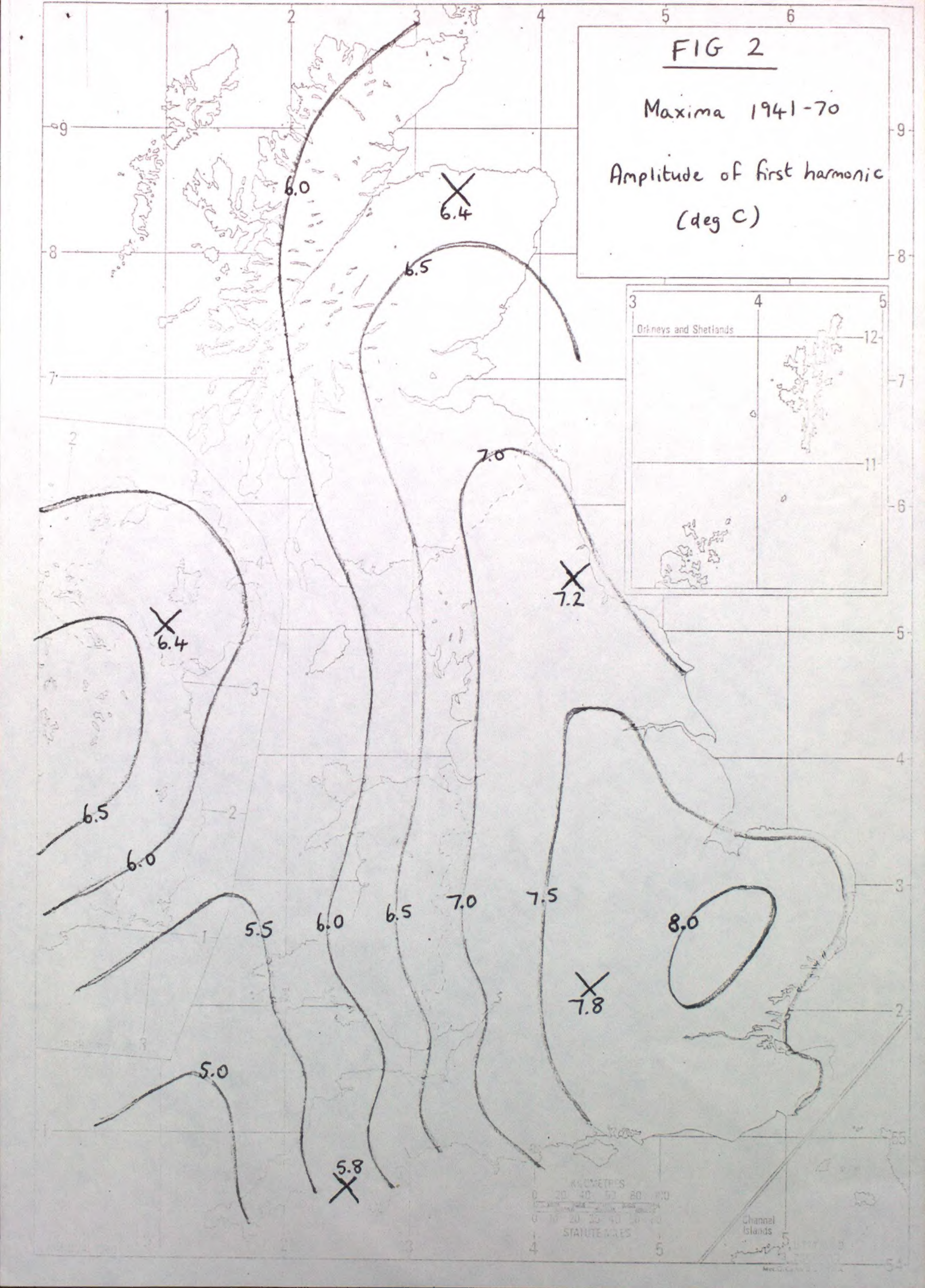
FIG 2
Maxima 1941-70
Amplitude of first harmonic
(deg C)
Orkneys and Shetlands
6.0
6.4
6.5
7.0
7.2
6.4
6.5
6.0
5.5
6.0
6.5
7.0
7.5
8.0
7.8
5.0
5.8
Channel Islands

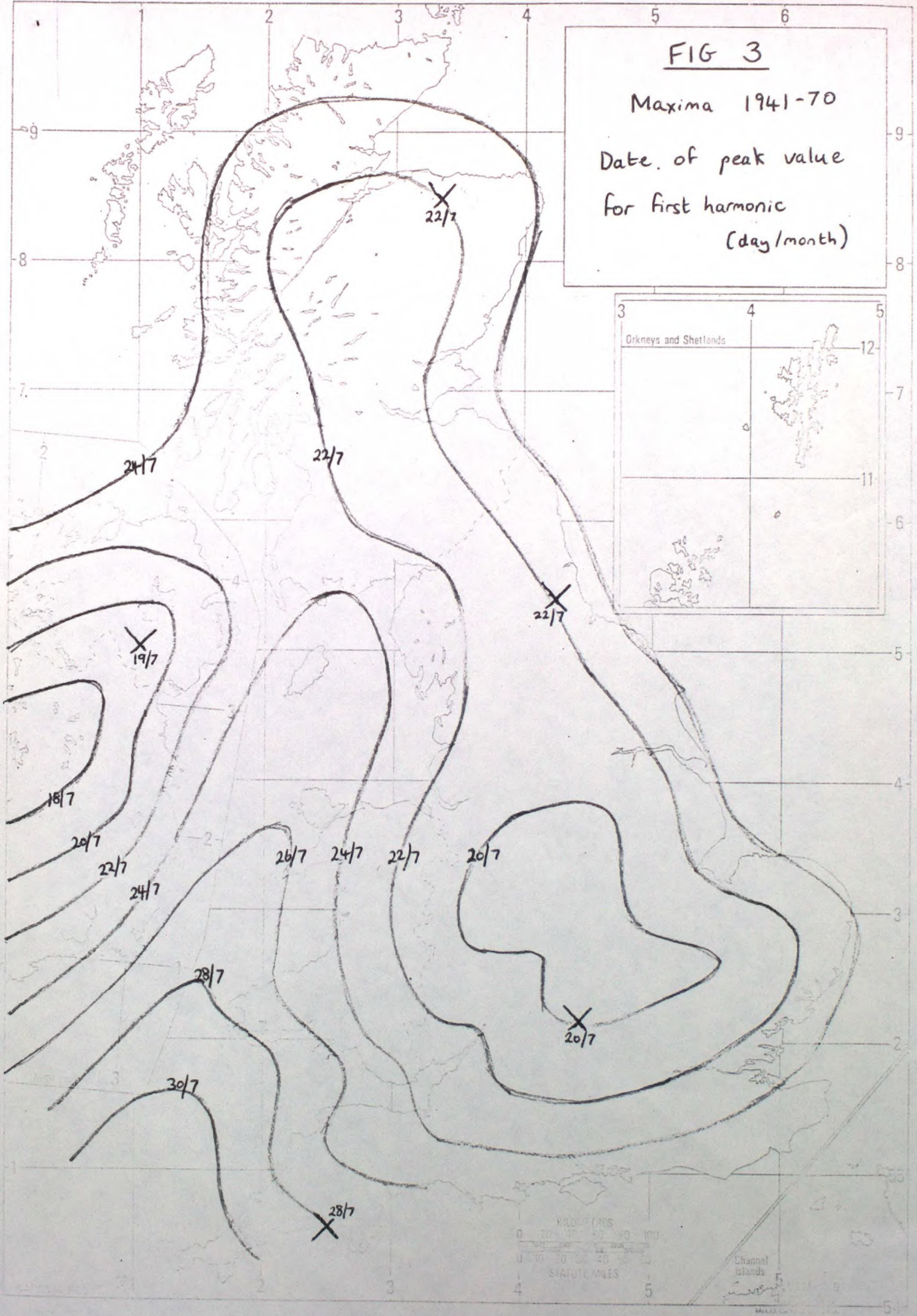
FIG 3
Maxima 1941-70
Date of peak value
for first harmonic
(day/month)
Orkneys and Shetlands
22/7
24/7
22/7
22/7
19/7
18/7
20/7
22/7
24/7
26/7
24/7
22/7
20/7
20/7
28/7
30/7
28/7
KILOMETRES
STATUTE MILES
Channel Islands

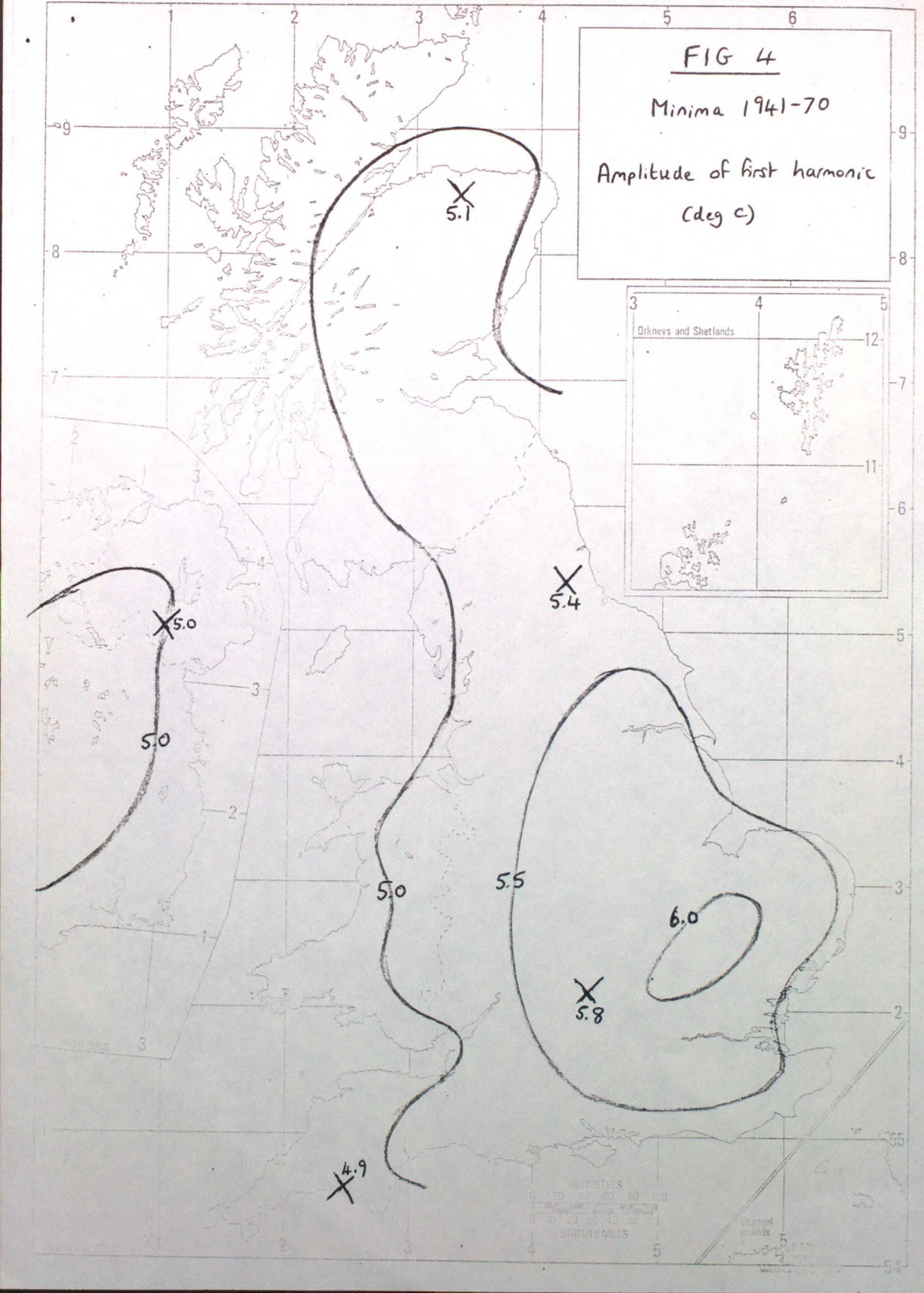
FIG 4
Minima 1941-70
Amplitude of first harmonic
(deg C)
Orkneys and Shetlands
5.1
5.4
5.0
5.0
5.0
5.5
6.0
5.8
4.9

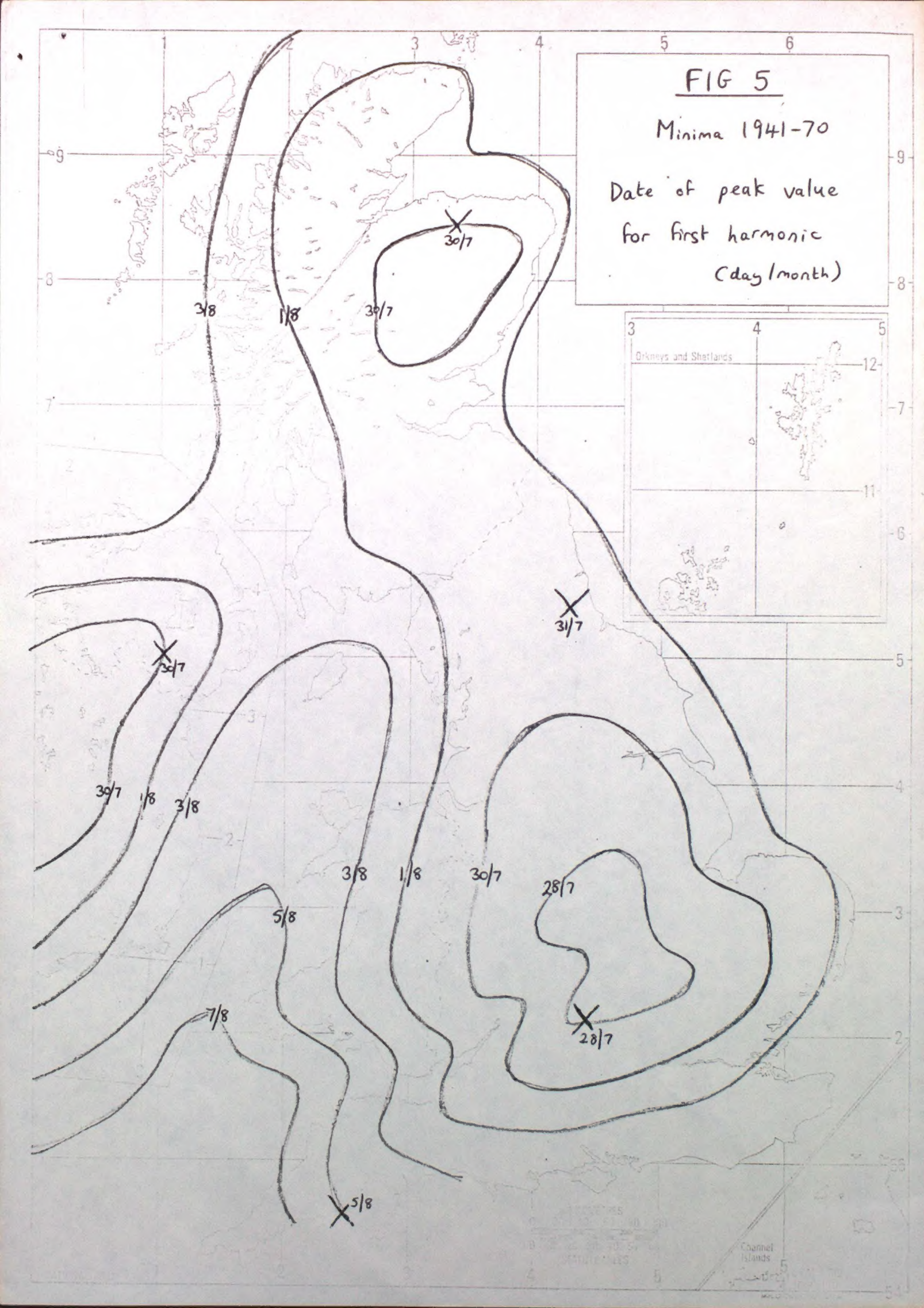
FIG 5
Minima 1941-70
Date of peak value
for first harmonic
(day/month)
Orkneys and Shetlands
30/7
3/8
1/8
30/7
31/7
30/7
1/8
3/8
3/8
1/8
30/7
28/7
5/8
7/8
28/7
5/8
Channel Islands